Quasi-Langevin method for shot noise calculation in single-electron tunneling

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(February 1, 2008)

Abstract

It is shown that quasi-Langevin method can be used for the calculation of the shot noise in correlated single-electron tunneling. We generalize the existing Fokker-Plank-type approach and show its equivalence to quasi-Langevin approach. The advantage of the quasi-Langevin method is a natural possibility to describe simultaneously the high ("quantum") frequency range.

73.40.Gk; 72.70.+m

Correlated single-electron tunneling [1] remains an attractive topic during last decade. Because in the systems of small-capacitance high-resistance tunnel junctions electrons tunnel almost as the classical particles, the most of experiments are well explained within the framework of "orthodox" theory [1] of single-electronics which is based on the classical master equation.

The theory of the shot noise in single-electron transistor [2] developed in Refs. [3,4] is also based on the master equation. The shot noise appears due to the randomness of the tunneling events. Despite the classical description of the system, the current in this theory should be treated as a kind of an operator because the current is caused by tunneling events which change the charge state of the system. This theory has been studied in detail in a number of subsequent publications - see, e.g., Refs. [5–11]. The first experimental confirmation of the theory has been obtained [12] in 1995, and other experiments are definitely coming because several groups has succeeded in the fabrication of the single-electron transistor operating at relatively high frequencies (hundreds of kHz) at which 1/f noise should become less than the shot noise.

Among the most noticeable developments of the shot noise theory in single-electronics after 1991, let us mention the following. It has been shown [9] that the noise due to the processes of Andreev reflection which transfer two electrons per tunneling event can be described by the same formalism with e substituted by $2e$. The shot noise in single-electron transistor with large level discreteness has been studied [13]. The shot noise theory has been applied to single-electron systems other than single-electron transistor [14,15]. Besides the noise in the "orthodox" frequency range $\omega \sim W/e^2R$ (where W is a typical energy and R is a typical resistance), the noise in the "quantum" frequency range $\omega \sim W/\hbar$ has been studied and the matching of the high and low frequency limits of two ranges has been proven [14]; however, the approach unifying both frequency ranges in one formalism has not been found.

The existing formalism of the shot noise in single-electron tunneling is of the Fokker-Plank type and is based on the deterministic master equation. However, generally more popular method in the study of the noise is the Langevin approach in which the random

term is introduced into the evolution equation. Three decades ago the Langevin approach was applied [16] to study the fluctuations in the nonequilibrium electron gas. In the present letter we show that a similar method can be used for the single-electron tunneling. (We call it quasi-Langevin because the regular Langevin method cannot be applied directly.) It is equivalent to existing formalism in the "orthodox" frequency range, but it naturally allows also the calculations in the "quantum" frequency range.

Let us start with the generalization of the existing Fokker-Plank type method to an arbitrary system consisting of voltage sources, capacitances and tunnel junctions with sufficiently large resistances ($R_j \gg R_K = h/e^2 \simeq 26\mathrm{k}\Omega$). For simplicity we do not consider Ohmic resistances. In this case the dynamics is governed by the matrix master equation

$$\dot{\sigma} = \mathbf{\Gamma}\sigma \tag{1}$$

where the vector σ_n is the probability to find the system in the charge state $n \equiv \{n_1, ...n_L\}$ (which is characterized by the numbers n_i of excess electrons in each of L internal nodes of the system) and

$$\Gamma_{mn} = \Gamma_{m\leftarrow n} - \delta_{mn}\sum_k \Gamma_{k\leftarrow n},\ \Gamma_{m\leftarrow n} = \sum_j \Gamma^j_{m\leftarrow n}, \tag{2}$$

where $\Gamma^j_{m\leftarrow n}$ are the corresponding tunneling rates and the summation over the junction number j is necessary when an electron can tunnel to (from) an internal node from (to) different external electrodes.

To find the mutual spectral density for two processes $X(t)$ and $Y(t)$ we can calculate first the correlation function $K_{XY}(\tau) = \langle X(t+\tau)Y(t)\rangle - \langle X\rangle\langle Y\rangle$ (brackets denote the averaging over time) and then take the Fourier transform $S_{XY}(\omega) = 2\int_{-\infty}^{+\infty} K_{XY}(\tau)\exp(i\omega\tau)\,d\tau$. If both X and Y are functions of the charge state n (for example, potential of a node) then the correlation function is given by the simple expression

$$K_{XY}(\tau) = \theta(\tau)\sum_{m,n} X(m)\,\sigma(\tau, m|n)\,Y(n)\,\sigma^{st}_n$$
$$+\,\theta(-\tau)\sum_{m,n} Y(m)\,\sigma(m, -\tau|n)\,X(n)\,\sigma^{st}_n - \langle X\rangle\langle Y\rangle\,, \tag{3}$$

where $\sigma(\tau, m|n)$ is the retarded Green's function of Eq. (1) being the probability to find the system in the state m at $t = \tau > 0$ if at $t = 0$ it was in the state n, $\langle X \rangle = \sum_n X(n)\sigma_n^{st}$, and σ_n^{st} is the stationary distribution, $\mathbf{\Gamma}\sigma^{st} = 0,\ \sum_n \sigma_n^{st} = 1$. (Notice that X and Y are classical variables, and their commutator is zero.)

However, if X and/or Y represent the current through a tunnel junction or in an external lead, the Eq. (3) should be modified. For example, if $X(t)$ is the current contribution corresponding to tunneling events $\Gamma^j_{m\leftarrow n}$ while $Y(t)$ corresponds to $\Gamma^{j'}_{m'\leftarrow n'}$, then (similar to Refs. [3,4])

$$\begin{aligned} K_{XY}(\tau)/\tilde{e}^j_{\pm}\tilde{e}^{j'}_{\pm} &= \theta(\tau)\,\Gamma^j_{m\leftarrow n}\,\sigma(\tau, n|m')\,\Gamma^{j'}_{m'\leftarrow n'}\sigma^{st}_{n'} \\ &\quad + \theta(-\tau)\,\Gamma^{j'}_{m'\leftarrow n'}\sigma(n', -\tau|m)\,\Gamma^j_{m\leftarrow n}\,\sigma^{st}_n \\ - \Gamma^j_{m\leftarrow n}\sigma^{st}_n\,\Gamma^{j'}_{m'\leftarrow n'}\sigma^{st}_{n'} &+ \delta_{mm'}\delta_{nn'}\delta_{jj'}\delta(\tau)\,\Gamma^j_{m\leftarrow n}\sigma^{st}_n\,. \end{aligned} \tag{4}$$

Here the last term is responsible for the high-frequency limit. The effective charges $\tilde{e}^j_{\pm}$ and $\tilde{e}^{j'}_{\pm}$ are determined by the direction of electron tunneling, $\tilde{e}^j_+ = -\tilde{e}^j_-$, and by the circuit capacitances [3,4] (so that $\tilde{e}^j = e$ only if the current through junction j is measured). Any current–current correlation function can be written as a sum of $K_{XY}(\tau)$ given by Eq. (4) over all possible transitions between charge states (such a sum is a counterpart of Eq. (3) in which the sum is written explicitly). The processes of cotunneling [17] in the simple master equation approximation [18] and fluctuations due to Andreev reflection [9] can be accommodated in this technique using the appropriate values of $\tilde{e}^j$ (for cotunneling the index j should obviously represent the set of junction numbers).

For the correlation functions when X is a current and Y is a function of the charge state (or vice versa), the recipe is the "combination" of Eqs. (3) and (4) while the term proportional to $\delta(\tau)$ is absent.

The expressions for spectral densities directly follow from Eqs. (3) and (4) because the Fourier transformation affects only the evolution operator $\sigma(\tau, m|n)$, and the corresponding Green's function in the frequency representation is simply obtained from Eq. (1)

$$\sigma(\omega, m|n) = \left[(-i\omega\mathbf{1} - \mathbf{\Gamma})^{-1}\right]_{mn}, \tag{5}$$

where $\mathbf{1}$ is the unity matrix. For example, Eq. (4) leads to the following spectral density

$$S_{XY}(\omega)/\tilde{e}^{j}_{\pm}\tilde{e}^{j'}_{\pm} = 2\,\Gamma^{j}_{m\leftarrow n}\left[(-i\omega\mathbf{1}-\mathbf{\Gamma})^{-1}\right]_{nm'}\Gamma^{j'}_{m'\leftarrow n'}\sigma^{st}_{n'} + 2\,\Gamma^{j'}_{m'\leftarrow n'}\left[(i\omega\mathbf{1}-\mathbf{\Gamma})^{-1}\right]_{n'm}\Gamma^{j}_{m\leftarrow n}\sigma^{st}_{n} + 2\,\delta_{nn'}\delta_{mm'}\delta_{jj'}\,\Gamma^{j}_{m\leftarrow n}\,\sigma^{st}_{n}. \quad (6)$$

This method [3,4] allows to calculate all spectral densities within the framework of "orthodox" theory, and at least for the single-electron transistor the numerical procedure is rather trivial because matrix $\mathbf{\Gamma}$ is three-diagonal and the calculation of Eq. (5) is straitforward [19]. There are some additional simplifications for $\omega \to 0$, however, in this paper we concentrate on the finite frequencies only. Notice that in the special case when there are only two charge states involved, the theory is equivalent to the noise calculations in resonant tunneling diode [20].

Now let us develop the Langevin-type approach. It is obvious that for the discrete random dynamics of the charge states of a single-electron circuit it is impossible to introduce the random term in Eq. (1) in a reasonable way. However, let us imagine the ensemble of M ($M \gg 1$) independent similar circuits, and let us average all magnitudes over this ensemble. Then the average (over time) currents and voltages will not change (due to ergodicity), but the spectral densities of fluctuations (second order magnitudes) will decrease M times. Hence, to calculate spectral densities of the initial system, we can take the leading ($\sim M^{-1}$) order of the spectral density of magnitudes averaged over the ensemble, in the thermodynamic limit $M \to \infty$. In contrast to the single system, the dynamics of the ensemble at $M \to \infty$ can be easily described by the Langevin approach because there are no longer step-like dependencies on time.

Now $M\sigma_n(t)$ which is the number of systems in the ensemble having the charge state n, can be considered as non-integer (because of $M \gg 1$) and its random dynamics can be described with the help of fictitious random term $\xi(t)$. In the "orthodox" framework (when we consider electron jumps as instantaneous events), $\xi(t)$ is δ-correlated (white), and its amplitude is defined by the Poissonian nature of the tunneling process. The recipe is the

following [16]: for each average flux $M\Gamma^j_{m\leftarrow n}\sigma^{st}_n$ (number of transitions per second) in the space of charge states, we should add in the master equation the random flux $\xi^j_{m\leftarrow n}(t)$ with the corresponding "seed" spectral density given by usual Schottky formula

$$\dot{\sigma}_m(t) = \sum_n \Gamma_{mn}\sigma_n(t) + \xi_m(t), \tag{7}$$

$$\xi_m(t) = \sum_{n,j} \xi^j_{m\leftarrow n}(t) - \xi^j_{n\leftarrow m}(t), \tag{8}$$

$$S_{\xi^j_{m\leftarrow n}\xi^{j'}_{m'\leftarrow n'}}(\omega) = 2M^{-1}\delta_{mm'}\delta_{nn'}\delta_{jj'}\Gamma^j_{m\leftarrow n}\sigma^{st}_n. \tag{9}$$

Notice that when there are fluxes in opposite directions ($m \leftarrow n$ and $n \leftarrow m$), we should apply $\xi(t)$ for each direction, so that the random flux does not vanish even if the net average flux is zero.

Because of the linearity of Eqs. (7)–(9) the final spectral densities of the averaged (over M) magnitudes are obviously proportional to $1/M$. Hence, rescaling to the single system can be done formally assuming $M = 1$ in Eqs. (7)–(9). So, instead of keeping M and rescaling at the final stage, we will use $M = 1$ in all equation below. We call this simple trick of considering first the large ensemble, writing the Langevin equation for it, and then returning to a single system, a quasi-Langevin approach.

Using the standard procedure we find the Fourier transform

$$\sigma_m(\omega) = \left[(-i\omega\mathbf{1} - \mathbf{\Gamma})^{-1}\right]_{mn}\xi_n(\omega). \tag{10}$$

Then for the occupation–occupation spectral density

$$\begin{aligned} S_{\sigma_m\sigma_n} = &\sum_{m'n'} \left[(-i\omega\mathbf{1} - \mathbf{\Gamma})^{-1}\right]_{mm'} \\ &\times \left[(i\omega\mathbf{1} - \mathbf{\Gamma})^{-1}\right]_{nn'} S_{\xi_{m'}\xi_{n'}}. \end{aligned} \tag{11}$$

Using Eq. (9) after simple algebra we obtain

$$\begin{aligned} S_{\sigma_m\sigma_n} = &\, 2\left[(-i\omega\mathbf{1} - \mathbf{\Gamma})^{-1}\right]_{mn}\sigma^{st}_n \\ &+ 2\left[(i\omega\mathbf{1} - \mathbf{\Gamma})^{-1}\right]_{nm}\sigma^{st}_m, \end{aligned} \tag{12}$$

which coincides with the result of Fokker-Plank approach (Fourier transform of Eq. (3) without X and Y factors).

The technique is similar for the current–current fluctuations. The case of Eqs. (4) and (6) corresponds to currents

$$X(t) = \tilde{e}^j_{\pm} \left[\Gamma^j_{m \leftarrow n} \sigma_n(t) + \xi^j_{m \leftarrow n}(t) \right],$$
$$Y(t) = \tilde{e}^{j'}_{\pm} \left[\Gamma^{j'}_{m' \leftarrow n'} \sigma_{n'}(t) + \xi^{j'}_{m' \leftarrow n'}(t) \right], \qquad (13)$$

and the straitforward (though rather lengthy) calculations using Eqs. (9) and (10) lead to Eq. (6).

The final expression for the current–occupation spectral density is

$$S_{X\sigma_k}(\omega)/\tilde{e}^j_{\pm} = 2\Gamma^j_{m \leftarrow n} \left[(-i\omega \mathbf{1} - \mathbf{\Gamma})^{-1} \right]_{nk} \sigma^{st}_k$$
$$+2 \left[(i\omega \mathbf{1} - \mathbf{\Gamma})^{-1} \right]_{km} \Gamma^j_{m \leftarrow n} \sigma^{st}_n , \qquad (14)$$

and it also coincides with the corresponding expression obtained in the Fokker-Plank technique.

Thus, we have proven that the Fokker-Plank method is equivalent to the quasi-Langevin method within the "orthodox" framework. However, in contrast to the former approach, the quasi-Langevin method easily allows generalization for the fluctuations in the "quantum" frequency range.

Let us remind that in "orthodox" theory [1] (from now on for simplicity we speak only about purely single-electron tunneling and do not consider cotunneling, Andreev reflection, etc.)

$$\Gamma = \frac{I_0(W/e)}{e(1 - \exp(-W/T))}, \; W = eV - e^2/2C_{eff} \qquad (15)$$

where $I_0(v)$ is the "seed" I-V curve of the junction (in the linear case $I_0(v) = v/R$), W is the energy gain due to tunneling, V is the voltage across the junction before the tunneling, and C_{eff} is the effective junction capacitance (which also accounts for the environment). The generalization of quasi-Langevin method is the substitution of Eq. (9) by more general equation [14]

$$S_{\xi^j_{m\leftarrow n}\xi^{j'}_{m'\leftarrow n'}}(\omega) = \delta_{mm'}\delta_{nn'}\delta_{jj'}[\tilde{\Gamma}^+ + \tilde{\Gamma}^-]\,\sigma_n^{st}\,,$$
$$\tilde{\Gamma}^\pm = \frac{I_{0,j}(W^j_{m\leftarrow n}/e \pm \hbar\omega/e)}{e\left[1-\exp\left((W^j_{m\leftarrow n} \pm \hbar\omega)/T\right)\right]}. \tag{16}$$

This equation can be considered as a generalization of the fluctuation-dissipation theorem and equations of Ref. [21] for the case of single-electron tunneling. It can be obtained within the standard tunneling hamiltonian technique averaging the quantum current–current correlator and then taking the Fourier transform.

Using this generalization one can see that at high frequencies, $\omega \gg \Gamma$, the occupation–occupation and occupation–current spectral densities vanish, while for the current–current spectral density Eq. (6) transforms into

$$S_{XY}(\omega) = \tilde{e}^j_\pm \tilde{e}^{j'}_\pm S_{\xi^j_{m\leftarrow n}\xi^{j'}_{m'\leftarrow n'}}(\omega), \tag{17}$$

(given by Eq. (16)) because the first terms of Eq. (13) are too slow to give a cotribution. This result coincides with the result of Ref. [14]. The advantage of the quasi-Langevin approach is the possibility to obtain spectral densities in the "orthodox" and "quantum" frequency ranges using the same formalism while in Ref. [14] they are necessarily treated on different footing.

In the "quantum" frequency range the current spectral density does not correspond directly to the available power because of the contribution from zero-point oscillations. The spectral density of the current calculated above can be considered as the power (within the unit bandwidth) going from the system to a small external resistance r (divided by r and the coupling factor α). To obtain the available power, we should subtract the power flow in the opposite direction which is the product of the voltage spectral density of the resistance $2\hbar\omega r\coth(\hbar\omega/2T_r)$ (the "receiver" temperature T_r can differ from T), the factor α, and the active conductance of the fluctuation source, which in the case corresponding to Eqs. (16)–(17) is given by [14]

$$\mathrm{Re}G(\omega) = \left((\tilde{e}^j_\pm)^2/2\hbar\omega\right)[\tilde{\Gamma}^+ - \tilde{\Gamma}^-] \tag{18}$$

(this expression obviously corresponds to the lowest order of photon-assisted processes). Hence, the contribution to be subtracted from $S_{\xi^j_{m\leftarrow n}}$ is equal to $2\hbar\omega\coth(\hbar\omega/2T_r)\mathrm{Re}G(\omega)$. Traditionally this contribution is called zero-point for $T_r = 0$. However, for finite temperature T a more natural choice of the "receiver" temperature is $T_r = T$. Notice that to get the total conductance $\mathrm{Re}G_t(\omega)$, Eq. (18) should be summed over all kinds of tunneling events.

The solid line 1 in Fig. 1 shows the numerical result for the spectral density $S_{II}(\omega)$ of the current in the external lead of the single-electron transistor consisting of two similar tunnel junctions with capacitances $C_1 = C_2 = C_\Sigma/2$ and resistances $R_1 = R_2 = R_\Sigma/2$ at $Q_0 = 0.3e$ and $T = 0.03e^2/C_\Sigma$. The frequency dependence in the "orthodox" frequency range is important at $f = \omega/2\pi \sim I/e$ while the "quantum" frequency dependence occurs at $f \sim e^2/C_\Sigma h$. Because the junction resistances are chosen sufficiently large, $R_\Sigma = 100R_K$, two frequency ranges are far away from each other (the ratio of typical frequencies is on the order of R/R_K). The line 2 shows the spectral density corresponding to the power available at zero-temperature receiver (zero-point contribution is subtracted, $T_r = 0$) while the line 3 shows the available power for $T_r = T$. Notice that the curve 3 differs considerably from curve 1 even in the "orthodox" frequency range because of thermodynamical reasons. The dashed lines 4 and 5 show the results for $T = 0$ (two lower curves obviously coincide). Notice the cusps at $hf = 0.05e^2/C_\Sigma$ which correspond to the extra energy above the Coulomb blockade and the abrupt vanishing of the available power above the frequency $f = W_{max}/h$ ($f = 0.45e^2/C_\Sigma h$ in this case) where W_{max} is the maximal energy gain among all possible tunneling events.

In the case $R_j \gg R_K$ the "orthodox" and "quantum" frequency ranges are far apart from each other because $\Gamma \sim W/eR \ll W/h$. The fact that the quasi-Langevin formalism describes the fluctuations for the whole frequency axis, suggests the possibility to use this method also when two frequency ranges are close to each other. However, this is impossible because when $R_j \sim R_K$ the master equation approach fails due to strong cotunneling processes [17]. The correct description of the noise in this case (as well as just calculation of the average currents) is still an open question despite an important progress for the average

currents [22].

In conclusion, we developed the quasi-Langevin method for the calculation of the fluctuations in single-electron tunneling. Its equivalence to the previous Fokker-Plank approach in the "orthodox" frequency range is proven. The advantage of the quasi-Langevin method is the natural generalization for the "quantum" frequency range.

The author thanks K. K. Likharev for attracting the interest to this problem and N. E. Korotkov for the basic introduction to the theory of fluctuations. The author is also grateful to D. V. Averin, S. V. Gantsevich, Sh. M. Kogan, V. V. Potemkin, G. Schön, A. S. Stepanov, and J. S. Tsai for the valuable discussions. The work was supported in parts by US AFOSR, Russian Fund for Basic Research, and Russian Program on Prospective Technologies for Nanoelectronics.

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FIGURES

FIG. 1. Curve 1 shows the spectral density of the current (in the external lead) through the symmetric single-electron transistor at $T = 0.03e^2/C_\Sigma$ as a function of frequency $f = \omega/2\pi$. The relatively large junction resistances provide cosiderable difference between "orthodox" and "quantum" frequency scales. The curve 2 shows the spectral density corresponding to the available power with zero-point cotribution subtracted ($T_r = 0$) while for the curve 3 the "receiver" is assumed to be at the same temperature ($T_r = T$). Dashed curves 4 and 5 demonstrate the results for $T = 0$. The average current $I = 0.106e/R_\Sigma C_\Sigma$ for $T = 0.03e^2/C_\Sigma$ while $I = 0.09e/R_\Sigma C_\Sigma$ for $T = 0$.

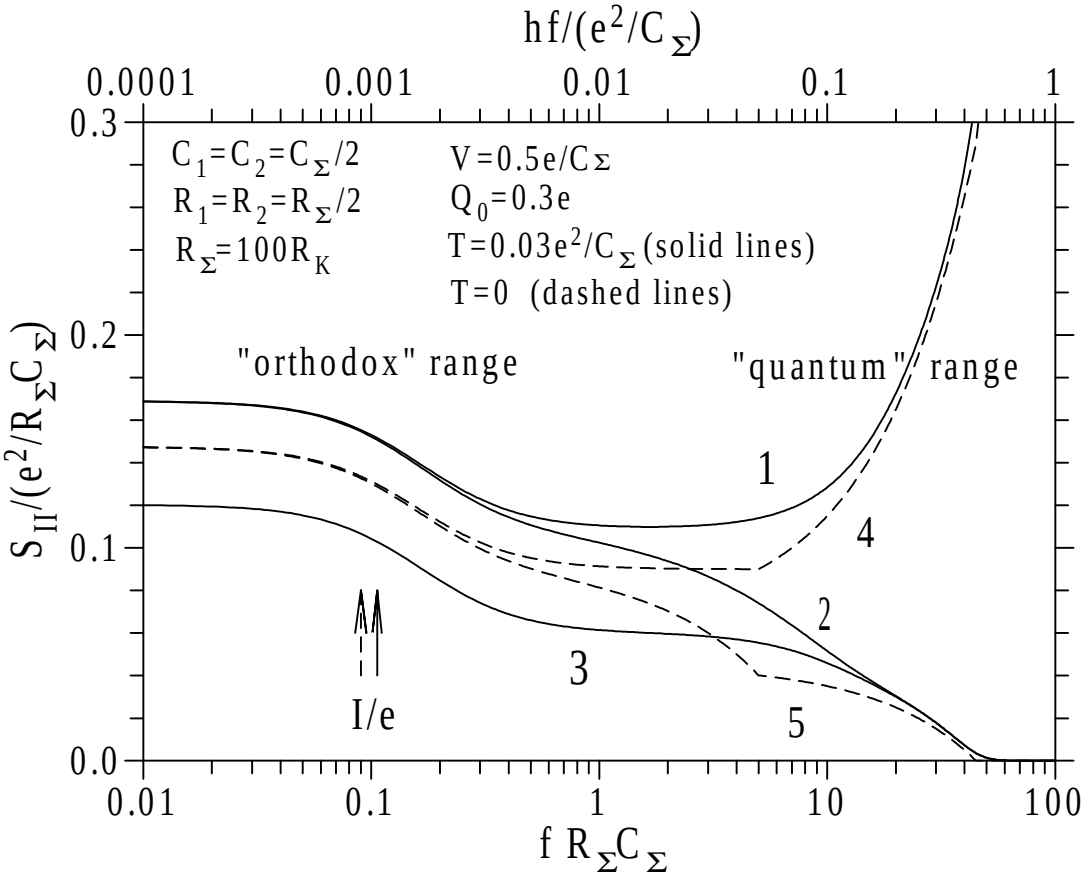
hf/(e²/CΣ)
0.0001
0.001
0.01
0.1
1
C1=C2=CΣ/2
R1=R2=RΣ/2
RΣ=100RK
V=0.5e/CΣ
Q0=0.3e
T=0.03e²/CΣ (solid lines)
T=0 (dashed lines)
"orthodox" range
"quantum" range
SII/(e²/RΣCΣ)
0.3
0.2
0.1
0.0
1
2
3
4
5
I/e
0.01
0.1
1
10
100
f RΣCΣ

Fig. 1